Indian Statistical Institute

Midterm Exam. 2025-2026

Functional Analysis, M.Math First Year

Time : 3 Hours   Date : 23.02.2025   Maximum Marks : 100   Instructor : Jaydeb Sarkar

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(i) Answer all questions. (ii) You may freely apply any of the theorems discussed in class.

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(1) (10 marks) Prove that $(C[0,1], \|\cdot\|_\infty)$ is a Banach space.

(2) (20 marks) Prove that any two norms on a finite-dimensional vector space are equivalent.

(3) (20 marks) Let S be a subset of a Hilbert space H. Prove that $S^{\perp\perp}$ is the smallest closed subspace of H containing S.

(4) (20 marks) Consider a sequence $\{\alpha_n\}_{n\geq 1} \subseteq \mathbb{C}$. Suppose

$$\lim_{N\to\infty} \sum_{n=1}^{N} a_n \alpha_n \in \mathbb{C},$$

for all $\{a_n\}_{n\geq 1} \in l^1$. Prove that

$$\{\alpha_n\}_{n\geq 1} \in l^\infty.$$

(5) (20 marks) Define

$$S = \Big\{ f \in L^2[-1,1] : \int_{-1}^{1} f = 0 \Big\}.$$

Prove that S is a closed subspace of $L^2[-1,1]$. Compute $S^\perp$, and the distance from f to S, where $f(t) = t^2$.

(6) (20 marks) Let X be a normed linear space, C be a closed subspace of X, and let F be a finite-dimensional subspace of X. Prove that $C + F$ is a closed subspace.